Spatio-temporal overload propagation of cascading failures

As the lifeline of human society, infrastructures including information networks, transportation networks as well as power grid networks are facing unprecedented challenge on their reliability from various threats. Starting from a localized failure, a cascading failure can propagate with a domino-like way, which may result in catastrophe such as those observed in many realistic networks. For instance, in March 2015, over half number of provinces of Turkey witnessed cascading blackout, which caused tremendous and inevitable damage in transport, air service as well as residential electricity consumption. Similar cascading failures also happen in information system. In Jan 2014, starting from the crash of servers of Tencent, over two-thirds of DNS servers in China resulted in collapse, which caused a major breakdown of web pages for many users. These cascading failures are mostly inconspicuous at their early stage, but wide spreading in an unexpected way, making it difficult to predict and mitigate.

For the failure causes, some of cascading failures are due to structural interdependency, while others result from overloads. Unlike the first kind of failures spread via direct contacts, overload failures usually propagate through invisible paths as a result of cooperative interactions in the system. In structural failures, the damage spreads directly via structural dependencies in the network connections that bind directly the elements rather than route the flow through alternative paths, which may cause some nodes to become overloaded and failed. The butterfly effect of cascading overload failures can cause catastrophic damage due to even a small local disturbance. Unfortunately, the knowledge of how the failures propagate in time and space is essential, but still missing. Therefore，the spatio-temporal overload propagation of cascading failures has become an important question in the study of complex system reliability.

Here we focus on the cascading overloads on spatially embedded networks caused by localized attacks, which are common in natural disasters and malicious attacks. To study its spatio-temporal properties, we propose two quantities: $\mathrm{r_c}(\mathrm{t})$ and $\mathrm{F_r}(\mathrm{t})$ at cascading step t. $r_c(t)$ is the average Euclidean distance, $F_r(t)$ is the number of node failures. $r_c(t)$ can help system regulators to set a 'firewall' at suitable locations before the failure arrives, and $F_r(t)$ can suggest the 'height' of the 'firewall'.

The results can be seen in the following figures. In Figure 1, we show snapshots of the simulated cascading failures, where the failures spread almost radially from the initial attack region and finally spread over the whole system. The nodes with spatial location closer to the initial failure begin to fail first and form approximately a 'ring of failures'. The ring begins to grow and expand with time until it reaches the system's boundary.

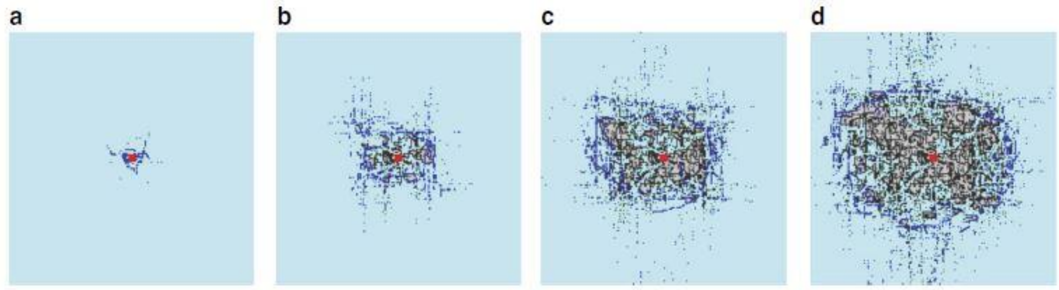

Fig 1. We demonstrate step 1, 3, 5 and 7 of the cascading failures on a 200*200 lattice. In each figure, the deep blue dots stands for the overloaded nodes in the current step, while the black ones are the nodes failed in the previous steps. The cyan dots are the functional nodes that did not fail.

As shown in Figure 2a, cascading failures spread during the cascade process with an almost constant speed. And Figure 2b notes that for different system sizes, the propagation size of failure $F_r(t)$ reaches the maximum at similar instants, which results from the higher velocity in larger system sizes. Figure 2c and 2d show the propagation velocity decreases with increasing tolerance, which implies that large tolerance can postpone the collapse of the system.

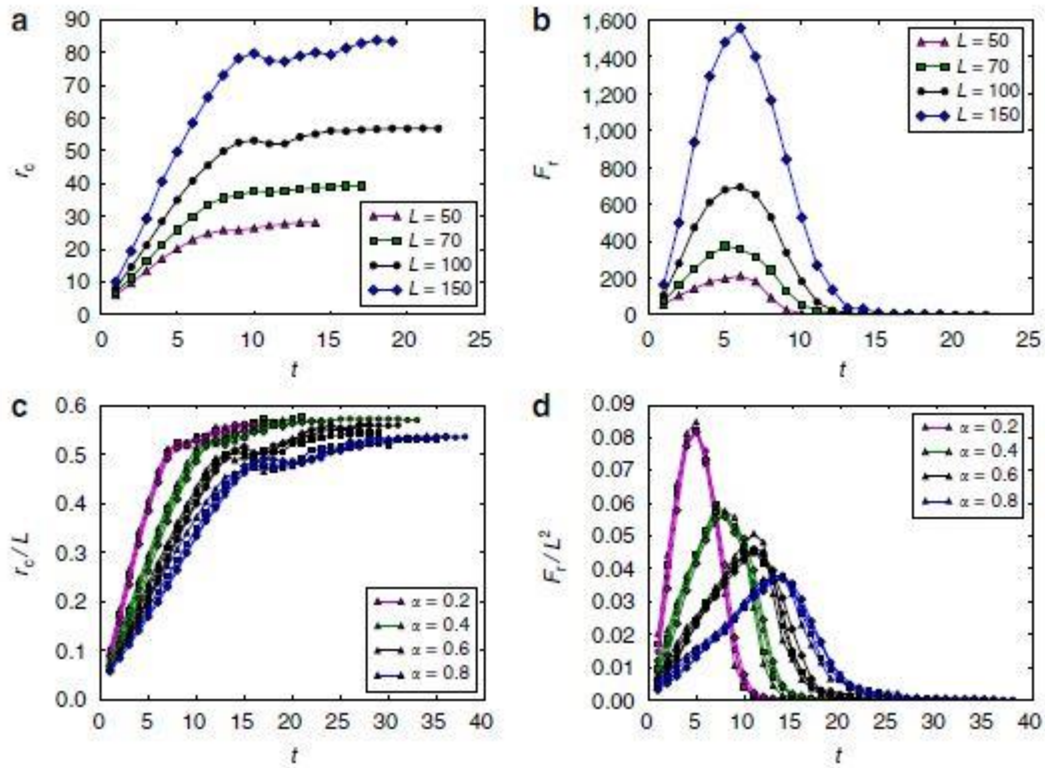

Fig 2. (a,b) are the spreading radius $r_c(t)$ of failures and amount of failures $F_r(t)$ at each step. (c,d) are the results of $r_c(t)$ and $F_r(t)$ scaled by the system size, including L=70 (triangle), L=80 (square), L=90 (circle) and L=100 (diamond),respectively.

Results we mentioned above are based on simulations, and we also have the theoretical analysis on cascading overloads. As shown in Figure 3, we compare the theory and simulation to prove that velocity can be well predicted by our theoretical framework with only one single correction for all the tolerance values. Both theory results and simulations suggest that for a given system size and a given tolerance, the size of initial failure does not influence the spreading velocity. Furthermore, the similar propagation velocity is also found in various model networks and real network structures.

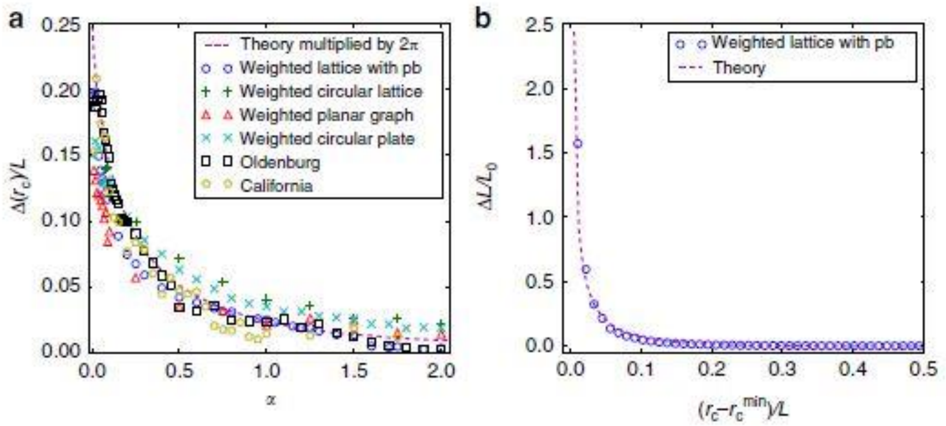

Fig 3. (a) Relative velocity as a function of tolerance in the theory, models as well as in real structures. The relative velocity is calculated in the linear regime of $\mathrm{r_c(t)}$, $\Delta(r_c)/L$, which decreases with α. (b) The average overload as a function of relative distance from the initial attack. The overload in the weighted lattice (circle symbol) and theory (dashed line) after the initial damage is shown.

The result suggests a possible bridge between systems with overload failures and networks with dependency links, which means networks with different characteristic length of dependency links can serve as a suitable model to describe cascading overload failures. This theory can be useful since overload models usually require heavy computations and are therefore limited to small systems, while dependency models require significantly less computations, and large-scale systems can be easily analyzed.

When a disturbance is detected in networks, the knowledge of spatio-temporal propagation properties of cascading failures is essential for predicting and mitigating the cascading failures. Meanwhile, realistic cascading failures are usually the result of the collective interactions between different processes including overloads and other system operation procedures, suggesting a long way towards fully revealing the hidden laws behind cascading overload failures.

Daqing Li, associate professor, school of reliability and system engineering, Beihang University, E-mail: daqingl@buaa.edu.cn or li.daqing.biu@gmail.com

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